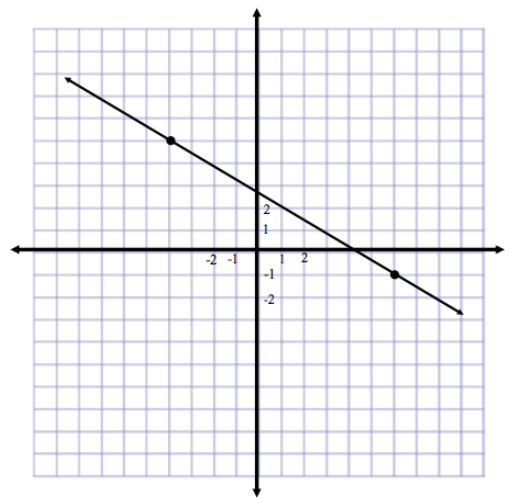


Name:

**Section 1: Linear Equations and Graphs of Linear Equations**

1. Find the slope of the line containing the given points.
  - a) (2, -1) and (6, 1)
  - b) (3, 7) and (-8, 7)
  - c) (9, 8) and (9, -7)
2. Use the results of 1 to write a description of each line.
  - a)
  - b)
  - c)
3. Write the equation of the line (in any form) that passes through the points (4, 3) and (-2, 6).
4. Write the equation of the line that is perpendicular to the line  $x - 3y = 2$  and passes through the point (0, 4).
5. Write the equation of the line graphed in the form  $Ax + By = C$ , where A, B and C are integers.
6. Consider the line  $L$  with equation  $y + 2x = 3$ . The line  $L_1$  is parallel to  $L$  and passes through the point (6, -4).
  - a. Find the gradient (slope) of  $L_1$ .
  - b. Find the equation of  $L_1$  in the form  $y = mx + b$ .
  - c. Find the  $x$  - coordinate of the point where line  $L_1$  crosses the  $x$ - axis.



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7. Two weeks after its birth, an animal weighed 13 kg. At 10 weeks this animal weighed 53 kg. The increase in weight each week is constant.
- Show that the relation between  $y$ , the weight in kg, and  $x$ , the time in weeks, can be written as  $y = 5x + 3$ .
  - Write down the weight of the animal at birth.
  - Write down the weekly increase in weight of the animal.
  - Calculate how many weeks it will take for the animal to reach 98 kg.

### **Section 2: Systems of Linear Equations and Matrices**

8. Solve the following system of equations using the substitution method. Describe the meaning of your result.

a) 
$$y = \frac{2}{3}x$$
$$x - 2y = -2$$

b) 
$$x - 5y = -5$$
$$-2x + 10y = 10$$

9. Solve the following system of equations using the combination method. Describe the meaning of your result.

a) 
$$2x - 5y = -10$$
$$3x + 5y = 20$$

b) 
$$x + 2y = 6$$
$$3x + 6y = 20$$

10. Let  $\mathbf{M} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ .

a. Find  $\mathbf{M}^2$ .

b. Show that  $\mathbf{M}^3 = \begin{pmatrix} 1 & 6 \\ 0 & 1 \end{pmatrix}$ .

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11. Let  $\begin{pmatrix} b & 3 \\ 7 & 8 \end{pmatrix} + \begin{pmatrix} 9 & 5 \\ -2 & 7 \end{pmatrix} = \begin{pmatrix} 4 & 8 \\ a & 15 \end{pmatrix}$ .

- a. Write down the value of  $a$ .
- b. Find the value of  $b$ .

Let  $3\begin{pmatrix} -4 & 8 \\ 2 & 1 \end{pmatrix} - 5\begin{pmatrix} 2 & 0 \\ q & -4 \end{pmatrix} = \begin{pmatrix} -22 & 24 \\ -9 & 23 \end{pmatrix}$ .

- c. Find the value of  $q$ .

12. Let  $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 0 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 18 \\ 23 \\ 13 \end{pmatrix}$ , and  $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ .

- a. Write down the inverse matrix  $A^{-1}$
- b. Consider the equation  $\mathbf{AX}=\mathbf{B}$ .
  - i. Express  $\mathbf{X}$  in terms of  $A^{-1}$  and  $\mathbf{B}$ .
  - ii. **Hence**, solve for  $\mathbf{X}$ .

### **Section 3: Sequences and Series and Binomial Theorem**

13. Let  $S_n$  be the sum of the first  $n$  terms of an arithmetic sequence whose first three terms are  $u_1$ ,  $u_2$  and  $u_3$ . It is known that  $S_1 = 7$  and  $S_2 = 18$ . ( $S_2$  is the sum of the first two terms).

- a. Write down  $u_1$ .
- b. Calculate the common difference of the sequence.
- c. Calculate  $u_4$

14. The first term of an infinite geometric sequence is 18, while the third term is 8. There are two possible sequences. Find the sum of each sequence.

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15. a. Consider the geometric sequence  $-3, 6, -12, 24, \dots$
- Write down the common ratio.
  - Find the 15<sup>th</sup> term.

Consider the sequence  $x - 3, x + 1, 2x + 8, \dots$

- b. When  $x = 5$ , the sequence is geometric.
- Write down the first three terms.
  - Find the common ratio
- c. Find the other value of  $x$  for which the sequence is geometric.
- d. For this value of  $x$ , find
- the common ratio;
  - the sum of the infinite sequence

16. Consider the expansion of  $(x^2 - 2)^5$ .

- Write down the number of terms in this expansion.
- The first four terms of the expansion in descending powers of  $x$  are

$$x^{10} - 10x^8 + 40x^6 + Ax^4 + \dots$$

Find the value of  $A$ .

17. Given that  $(3 + \sqrt{7})^3 = p + q\sqrt{7}$  where  $p$  and  $q$  are integers, use the binomial theorem to find the values of  $p$  and  $q$ .

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18. Find the term containing  $x^3$  in the expansion of  $(2 - 3x)^8$ .

**Section 4: Functions and Inverses**

19. Let  $f(x) = 2x + 1$  and  $g(x) = x^2 - 4$ .

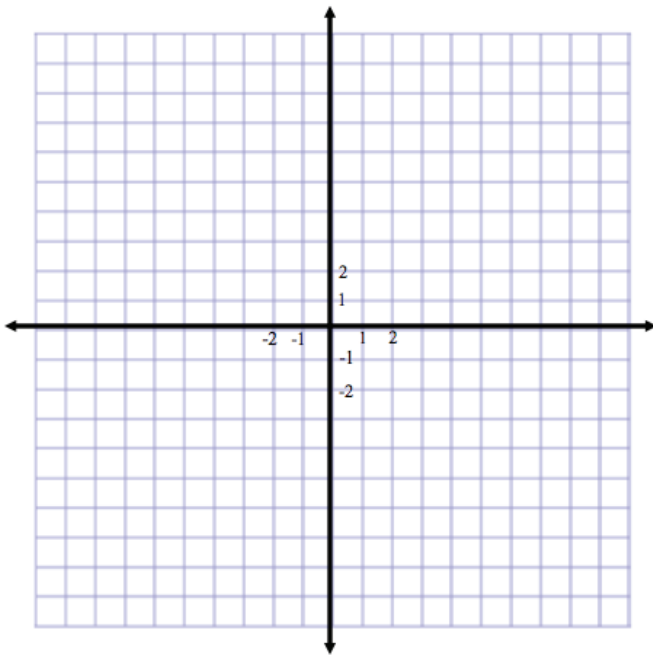
Find

- a.  $f^{-1}(x)$ ;
  - b. Graph  $g(x)$  for  $0 \leq x \leq 4$  and  $g^{-1}(x)$  on the same set of axes. Write the equation for  $g^{-1}(x)$ .
  - c.  $(g \circ f)(-2)$  (may be known as  $g(f(-2))$ )
  - d.  $(f \circ g)(x)$
20. The equation  $x^2 - 2kx + 1 = 0$  has two distinct real roots. Find the set of all possible values of  $k$ .
21. The function  $f$  is given by  $f(x) = x^2 - 6x + 13$ , for  $x \geq 3$ .
- a. Write  $f(x)$  in the form  $(x - a)^2 + b$ .
  - b. Find the inverse function  $f^{-1}$ .
  - c. State the domain of  $f^{-1}$ .
22. The function  $f$  is given by  $f(x) = e^{(x-1)} - 8$ .
- a. Find  $f^{-1}(x)$ .
  - b. Write down the domain of  $f^{-1}(x)$ .

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23. Consider the function  $f(x) = \frac{8}{x-5} + 4$ ,  $x \neq 5$

- a. Write down the **equation** of
  - i. the vertical asymptote;
  - ii. the horizontal asymptote.
  
- b. Find the
  - i.  $y$  – intercept;
  - ii.  $x$  – intercept.
  
- c. Sketch the graph of  $f$ , clearly showing the above information.



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### **Section 5: Exponents and Logarithms**

24. Without using your calculator, find the **exact** solution of the equation  $9^{2x} = 27^{(1-x)}$ .
25. a. Given that  $\log_3 x - \log_3(x - 5) = \log_3 A$ , express  $A$  in terms of  $x$ .
- b. Hence or otherwise, solve the equation  $\log_3 x - \log_3(x - 5) = 1$ .
26. A machine was purchased for \$10 000. Its value  $V$  after  $t$  years is given by  $V = 10000e^{-0.3t}$ . The machine must be replaced at the end of the year in which its value drops below \$1500. Determine in how many years the machine will need to be replaced.
27. Let  $a = \log x$ ,  $b = \log y$ , and  $c = \log z$ . Write  $\log\left(\frac{x^2\sqrt{y}}{z^3}\right)$  in terms of  $a$ ,  $b$  and  $c$ .
28. Solve the following equations:
- a.  $\ln(x + 2) = 3$
- b.  $10^{2x} = 500$

### **Section 6: Statistics**

29. Let  $a$ ,  $b$ ,  $c$  and  $d$  be integers such that  $a < b$ ,  $b < c$ , and  $c = d$ .

The mode of these four numbers is 11.

The range of these four numbers is 8.

The mean of these four numbers is 8.

Calculate the value of each of the integers  $a$ ,  $b$ ,  $c$ ,  $d$ .

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30. The table below shows the marks gained in a test by a group of students

Mark	1	2	3	4	5
Number of students	5	10	$p$	6	2

The median is 3 and the mode is 2. Find the **two** possible values of  $p$ .

31. The 45 students in a class each recorded the number of whole minutes,  $x$ , spent doing experiments on Monday. The results are  $\sum x = 2230$  (recall that capital sigma means “sum” so that means the sum of all  $x$  values)

- a. Find the mean number of minutes the students spent doing experiments on Monday.

Two new students joined the class and reported that they spent 37 minutes and 30 minutes respectively.

- b. Calculate the new mean including these two students.