

PARABOLA INVESTIGATION

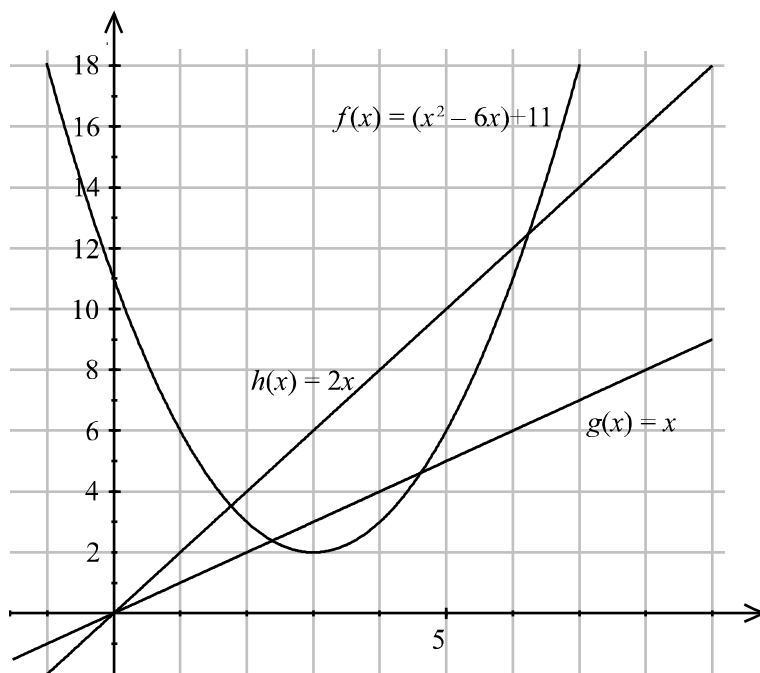
HL TYPE I

Description

In this task, you will investigate the patterns in the intersections of parabolas and the lines $y = x$ and $y = 2x$. Then you will be asked to prove your conjectures and to broaden the scope of the investigation to include other lines and other types of polynomials.

Method

1. Consider the parabola $y = (x-3)^2 + 2 = x^2 - 6x + 11$ and the lines $y = x$ and $y = 2x$.
 - Using technology find the four intersections illustrated on the right.
 - Label the x -values of these intersections as they appear from left to right on the x -axis as $x_1, x_2, x_3,$ and x_4 .
 - Find the values of $x_2 - x_1$ and $x_4 - x_3$ and name them respectively S_L and S_R .
 - Finally, calculate $D = |S_L - S_R|$.



2. Find values of D for other parabolas of the form $y = ax^2 + bx + c$, $a > 0$, with vertices in quadrant 1, intersected by the lines $y = x$ and $y = 2x$. Consider various values of a , beginning with $a = 1$. Make a conjecture about the value of D for these parabolas.
3. Investigate your conjecture for any real value of a and any placement of the vertex. Refine your conjecture as necessary, and prove it. Maintain the labeling convention used in parts 1 and 2 by having the intersections of the first line to be x_2 and x_3 and the intersections with the second line to be x_1 and x_4 .
4. Does your conjecture hold if the intersecting lines are changed? Modify your conjecture, if necessary, and prove it.
5. Determine whether a similar conjecture can be made for cubic polynomials.
6. Consider whether the conjecture might be modified to include higher order polynomials.