

Summer 2011 Assignment: Part A (*mandatory*—counts as a Problem Set)

Points = \_\_\_\_\_ out of 264

## SHOW ALL LOGICAL STEPS

- Write all work in pen or pencil by hand in the space provided. If more space is needed for a particular problem, write the continuation of your work on a separate sheet of paper that you will staple to this packet in sequence. Indicate that your work continues, and write your answer in the blank provided on the original.
- You may consult textbooks and other reference material, as well as your classmates. *There may be some unfamiliar problems where you must research the content or concepts first, and you are expected to do so.*
- A calculator may be used and may be required for some answers, while showing all work that sets up, supports, and reflects what the calculator executes.
- Each distinct numbered or lettered problem is worth 3 points.
- In some cases the steps are the answer; in other cases they support and lead to your simplified answer.
- In either situation, **CLEARLY SHOW ALL LOGICAL STEPS** as appropriate. If in doubt, write more.
- Thus, **WRITE KEY STEPS EXPLICITLY, SEQUENTIALLY, ORDERLY** (using distinct rows and/or columns), **AND LEGIBLY**, even when a calculator is used for one or more steps (and write “USED GDC” for those steps).
- **WRITE your SIMPLIFIED ANSWER IN THE BLANK PROVIDED.**
- **SKETCH and FULLY LABEL your GRAPH IN THE SPACE PROVIDED** as appropriate. Use graph paper as needed or desired, with the same full labeling and referencing to/identification of such graphs.
- Responses without supporting work may receive no credit.
- Unless otherwise specified, provide exact answers or answers to three significant figures.

1. Solve each equation. Give all roots, real or otherwise.

a.  $x^2 - 6x = -18$

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b.  $4x^2 - 12x + 7 = 0$

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c.  $\sqrt{2x+1} = \sqrt{x-3} + 2$

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2. Write an equation of the form  $f(x) = ax^2 + bx + c$  in which  $f(0) = -6$ ,  $f(2) = -3$ , and  $f(4) = 2$ .

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3. Consider the function  $P(x) = x^3 + 2x^2 - 5x - 6$ .

a. One root of  $P(x) = x^3 + 2x^2 - 5x - 6$  is  $x = -1$ . Find the other roots.

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b. Sketch the graph of  $y = x^3 + 2x^2 - 5x - 6$ .

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4. Find the maximum area of a rectangle whose perimeter is 100 cm.

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5. Find all complex roots of  $x^3 - 3x^2 + 4x - 12 = 0$ .

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6. Find a cubic equation with integral coefficients and roots  $-\frac{1}{3}$  and  $2 + 2i$ .

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7. Solve and graph each inequality on a number line. Use interval notation for solution sets.

a.  $|x - 5| > 12$

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b.  $(x - 2)^2(x + 3)(x - 1) \geq 0$

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8. Graph the solution set of the system of inequalities  $x \geq 0$  and  $y \leq 1 - x^2$ .

9. Give the domain, range, and zeroes of  $f(x) = \sqrt{9 - x^2}$ .

10. Graph  $g(x) = \begin{cases} x+1 & \text{if } x < -1 \\ 1-x^2 & \text{if } x \geq -1 \end{cases}$ . Find the range and zeroes of  $g$ .

11. Let  $f(x) = x^2 + 2x$  and  $g(x) = x + 2$ . Find:

a.  $(f + g)(x)$

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b.  $(f - g)(x)$

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c.  $(f \cdot g)(x)$

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d.  $\left(\frac{f}{g}\right)(x)$

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e.  $(f \circ g)(x)$

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f.  $(g \circ f)(x)$

12. Determine whether the graph of  $x^2 - xy = 4$  has symmetry in:

a. the  $x$ -axis

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b. the  $y$ -axis

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c. the line  $y = x$

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d. the origin

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13. Give a general expression for the value of a piece of property  $t$  years from now if its current value is \$150,000 and property values are increasing at the rate of 9% per year.

14. Solve  $\log_5 x + \log_5 (x - 4) = 1$ .

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15. State whether each function  $f$  has an inverse function in the following cases. If  $f^{-1}$  exists, find a rule (algebraic expression) for  $f^{-1}(x)$  and show that  $f(f^{-1}(x)) = f^{-1}(f(x)) = x$  for each such case.

a.  $f(x) = 8 - 2x$

b.  $f(x) = x^3 - x^2$

c.  $f(x) = \frac{1}{x-2}$

16. Solve the system 
$$\begin{cases} x^2 + y^2 = 25 \\ 2x^2 - 3y = 6 \end{cases}$$

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17. A sector of a circle has area  $8.1\pi \text{ cm}^2$  and central angle  $36^\circ$ . Find the radius and arc length.

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18. Let  $\cot x = \frac{5}{12}$ , where  $\pi < x < \frac{3\pi}{2}$ . Find the values of the other five trigonometric functions.

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19. Let  $y = \tan x$ .

a. Sketch the graph of  $y = \tan x$  for  $-\pi \leq x \leq 2\pi$ .

b. Give the domain, range, and period of the tangent function.

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20. Find the inclination of the line  $3x - 4y = 6$  to the nearest degree.

21. Sketch the graph of each equation.

a.  $y = -2 \sin \frac{x}{2}$

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b.  $y + 1 = 3 \cos \frac{\pi}{4}(x - 2)$

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22. Simplify  $\frac{1 - \cos \theta}{\sin^2 \theta} + \tan^2 \theta - \sec^2 \theta$ .

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23. A parallelogram has sides of lengths 4 cm and 9 cm with an included angle of  $40^\circ$ . Find the area of the parallelogram to the nearest tenth of a square centimeter.

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24. Two forest rangers at observation posts  $A$  and  $B$ , 10 mi apart, spot a fire at point  $F$ . The forest rangers know that  $\angle FAB = 110^\circ$  and  $\angle FBA = 25^\circ$ . To the nearest tenth of a mile, how far is the fire from the nearer observation post?

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25. Find  $\tan\left(\frac{3\pi}{4} + \theta\right)$  when  $\tan\theta = -\frac{1}{2}$ .

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26. Suppose  $\alpha$  is an acute angle and  $\cos\alpha = \frac{1}{3}$ . Find  $\sin 2\alpha$  and  $\cos\frac{1}{2}\alpha$ .

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27. Give polar coordinates for the point  $(-1, \sqrt{3})$ . Then give two other pairs of polar coordinates for the same point.

28. Give the rectangular coordinates for the point  $(-6, 30^\circ)$ .

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29. Let  $r = 4 \cos \theta$ .

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a. Sketch the polar graph of  $r = 4 \cos \theta$ .

b. Give a rectangular equation of this graph.

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30. Sketch the polar graph of  $r = 1 - \sin \theta$  and identify the graph.

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31. Let  $z_1 = 1 - i\sqrt{3}$  and  $z_2 = -1 + i\sqrt{3}$ .

a. Express  $z_1 z_2$  in rectangular form.

b. Express  $z_1$ ,  $z_2$ , and  $z_1 z_2$  in polar form.

c. Show that your answers for  $z_1 z_2$  in parts (a) and (b) agree.

32. Let  $z = -2 - 2i$ . Use de Moivre's theorem to find  $z^6$ .

33. Find the cube roots of  $1 - i$ . Write your answers in polar form.

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34. Find the distance of a ship in miles from its starting point if it sails 3 mi on a heading of  $040^\circ$  followed by sailing 8 mi on a heading of  $100^\circ$ . Illustrate the scenario with an appropriate diagram.

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35. Let the  $n$ th term of a geometric sequence be defined by  $t_n = \left(\frac{1}{3}\right)^{n-1}$ .

a. Show that the  $n$ th partial sum is  $S_n = \frac{3}{2} \left[ 1 - \left(\frac{1}{3}\right)^n \right]$ .

b. Find  $\lim_{n \rightarrow \infty} S_n$ .

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36. Recall that  $e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$ , and note that  $\left(1 + \frac{k}{n}\right)^n = \left(1 + \frac{1}{\frac{n}{k}}\right)^n = \left[\left(1 + \frac{1}{\frac{n}{k}}\right)^{\frac{n}{k}}\right]^k$ .

a. Show that  $\lim_{n \rightarrow \infty} \left(1 + \frac{3}{n}\right)^n = e^3$ .

b. Evaluate  $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{2n}\right)^n$ .

37. If  $\mathbf{u} = \langle 2, -1 \rangle$  and  $\mathbf{v} = \langle -3, 4 \rangle$ , find:

a.  $2\mathbf{v}$

b.  $3\mathbf{u} + \mathbf{v}$

c.  $|\mathbf{u} - \mathbf{v}|$

38. Consider the point  $(-2, -4)$  and the vector equation of the line  $\langle x, y \rangle = \langle 1, 3 \rangle + t \langle -2, 5 \rangle$ .

a. Find a vector equation of the line passing through that point and parallel to that line.

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b. What are the corresponding parametric equations for this line?

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39. At time  $t$ , the position of an object with constant velocity is given by the parametric equations  $x = 1 + 2t$  and  $y = -1 - 4t$ .

a. What are the velocity and speed of the object?

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b. When and where does it cross the line  $2x - y = 11$ ?

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40. Given the points  $A(1,3)$ ,  $B(-2,-5)$ , and  $C(6,-4)$ , find the measure of  $\angle B$  to the nearest tenth of a degree.

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41. Write vector and parametric equations for the line containing the points  $A(2,0,5)$  and  $B(-3,-1,4)$ .

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42. Given the points  $A(1,7,3)$  and  $B(-3,5,5)$ , find a Cartesian equation of the plane perpendicular to  $\overline{AB}$  at its midpoint,  $M$ .

43. Evaluate:  $\begin{vmatrix} 10 & 4 & 3 \\ 0 & 2 & 5 \\ -1 & -7 & 0 \end{vmatrix}$

44. Use Cramer's rule to solve:  $\begin{cases} 4x - 5y = 2 \\ 5x - 7y = 1 \end{cases}$

45. Find a vector perpendicular to the plane containing the points  $P(1, 0, 5)$ ,  $Q(-4, 3, 2)$ , and  $R(1, -1, -4)$ . Then find an equation of this plane.

46. Evaluate the given limit or state that the limit does not exist if such is the case. If the sequence for the given general term approaches  $\infty$  or  $-\infty$ , say so.

a.  $\lim_{n \rightarrow \infty} \frac{3n^2 + 2n + 7}{n^2 - 9}$

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b.  $\lim_{n \rightarrow \infty} \frac{5n^3 - 2n}{n^2 + 1}$

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c.  $\lim_{n \rightarrow \infty} [1 + (-1)^n]$

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47. Express the repeating decimal  $0.345345345\dots$  as a rational number.

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48. Solve this matrix equation for  $X$  : 
$$\begin{pmatrix} 3 & 4 \\ -3 & -3 \end{pmatrix} X - \begin{pmatrix} 2 & -5 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 4 \\ -2 & 1 \end{pmatrix}$$

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49. Write the following system of equations as a single matrix equation, and then solve the system by solving the matrix equation:

$$\begin{aligned} 3x + 5y &= 4 \\ x + 6y &= 10 \end{aligned}$$

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50. Evaluate each limit or state that the limit does not exist if such is the case. If the expression approaches  $\infty$  or  $-\infty$ , say so.

a. 
$$\lim_{x \rightarrow 1} \frac{2x^2 - x - 1}{2x^2 - 3x + 1}$$

b.  $\lim_{x \rightarrow \infty} \frac{3x^2 + 2}{2x^2 + x}$

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c.  $\lim_{x \rightarrow 2^+} \frac{x+1}{2-x}$

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51. Sketch the graph of each of the following functions. Along with labeling other key features, show vertical and horizontal asymptotes and  $x$ -intercepts.

a.  $y = \frac{x+2}{1-x}$

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b.  $y = \frac{x}{x^2 - 4}$

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c.  $y = \frac{x+1}{x^2+1}$

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52. Find the derivative of each function. Express the derivative without using fractional or negative exponents.

a.  $f(x) = 3x^2 - x + \frac{4}{x} - \frac{1}{2x^2}$

b.  $f(x) = \sqrt{2x} - \frac{2}{\sqrt{x}}$

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53. Find the slope of the curve  $y = 1 - 2x^2$  at the point  $P(-1, -1)$ .

54. Use the derivative of  $f(x) = 3x^4 + 4x^3 - 12x^2 + 6$  to sketch the graph of  $f(x)$ . Along with labeling other key features, identify any local maximum and minimum points.

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55. From a square piece of cardboard 10 inches on a side, a box with a square base and an open top is to be constructed by cutting out four squares at the corners and folding up the sides. What should the dimensions of the box be in order to maximize the volume of the box?

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56. A particle moves along a line so that its distance in feet to the right of its starting point after  $t$  seconds is  $s(t) = 8t - t^2$ . Find the times during which the particle (a) moves to the right and (b) moves to the left.

Summer 2011 Assignment: Part B (*optional*—counts as a Test)

In an essay of approximately 350 to 500 words (not magic numbers) plus algebraic statements and graphical illustrations, formally define what it means for a function to be continuous at a point, and then define a continuous function. Note that there are two different approaches to—or degrees of—this formality, and I want you to address both. Next, informally define continuity, and describe it and some of its key attributes in your own words. Use examples as may be helpful, including but not limited to algebraic statements and graphical representations, but examples should help in the development of your description of continuous functions generally and not be the focus of your discussion. Write for a reader who is well versed in algebra and has completed an introductory study of limits, but may be encountering the concept of continuous functions for the first time when reading your brief essay.

Use any resource you prefer; use quotation marks for whatever you quote or import; cite whatever you consult, whether you quote it or not; and write logically, clearly, and fluidly. Compose, save, and transmit your essay in a Word or PDF document; use MathType (available at <http://www.dessci.com/en/products/MathType/>) or something comparable if possible for mathematical notation and statements; and import images from your TI-84 or TI-89 or graphing software of your choice (many fine programs are available for free, e.g., WZGrapher (available at [http://www.walterzorn.com/grapher/grapher\\_app.htm](http://www.walterzorn.com/grapher/grapher_app.htm))).

You may freely consult your classmates as well as any other resource, but the work you submit must be your own. You may consult me for guidance, but only after you have thought about the task and concepts at some length and have done research on your own; your questions to me must reflect your research and your thinking.